Technical Comments

G 80-067 refers to

Comment on "Dynamics of a Flexible Body in Orbit"

(G 80-013 (P))

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IN Ref. 1, it is concluded that the flexural vibrations of a slender straight beam stabilized by gravity gradient may be unstably coupled with rigid-body pitch motion if the ratio of the flexural vibrational frequency to the orbital angular velocity, ω_n/ω_c, is near unity.

This conclusion is incorrect in at least two respects. In the first place, the frequency of the lowest bending mode is considerably higher than the orbiting angular velocity. In the second place, $\omega_n/\omega_c \simeq 1$ is not the condition for instability.

A lower limit for flexural vibration frequencies can be established as follows. In the limit, as the beam becomes more slender, its equations of motion in the pitch direction approach that of a tensioned string. If the mass distribution is uniform, then in the limit of slenderness the vibration frequencies approach

$$\omega_n = \omega_c \sqrt{\frac{3n(n+1)}{2}} \qquad (n=0,1,2,3,\dots) \tag{1}$$

The rigid-body mode corresponds to $n=1$ and has the frequency $\omega_n = \sqrt{3}\omega_c$, as noted in Eq. (5) of Ref. 1. The lower limit of the first flexural mode, corresponding to $n=2$, is $\omega_n = 3\omega_c$. Thus, the ratio ω_n/ω_c is at least equal to 3 for uniform gravity gradient stabilized beams.

This is not to say that the general class of instability studied in Ref. 1 (parametric resonance) cannot occur in the presence of a gravitational potential. Study of the Mathieu-Hill equation indicates small unstable regions near *every* integer and half-integer value of the ratio of natural frequency to the frequency of parametric excitation. Direct application of this observation to Eq. (6) of Ref. 1 shows that the frequencies of parametric excitation are the pitch frequency and twice the pitch frequency. Thus, the centers of the unstable regions are near the half-integer and integer harmonics of the pitch frequency, and not near the orbital frequency as stated in Ref. 1.

References

[1] Kumar, V.K. and Bainum, P.M., "Dynamics of a Flexible Body in Orbit," *Journal of Guidance and Control*, Vol. 3, Jan.-Feb. 1980, pp. 90-92.

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G 80-068

Reply by Authors to R.H. MacNeal

(G 80-013 (P))

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THE authors would like to acknowledge the interest that Mr. MacNeal has shown in our recent Engineering Note.[1] Mr. MacNeal's comments are apparently based on the following model of a tensioned string in orbit which he claims to be a suitable model as the beam becomes more slender. The string, of length $2l$, is assumed to be rotating at a uniform angular velocity of ω_c (orbital angular velocity). At any section located at a distance ξ from the center of the string, the tension due to the combined centrifugal and gravity-gradient effects is given by

$$T(\xi) = (3/2)\omega_c^2(l^2 - \xi^2) \tag{1}$$

The free transverse vibrations of a string with tension varying along the length of the string may be described by the following partial differential equation[2]:

$$\frac{\partial}{\partial \xi}\left[T(\xi)\frac{\partial y}{\partial \xi}(\xi,t)\right] = \rho\frac{\partial^2 y}{\partial t^2}(\xi,t) \tag{2}$$

where

y = transverse displaement of the string
ρ = mass per unit length of the string
t = time

After substitution of $T(\xi)$, given in Eq. (1), into Eq. (2), the following partial differential equation is obtained:

$$(1-x^2)\frac{\partial^2 y}{\partial x^2} - 2x\frac{\partial y}{\partial x} = \frac{2}{3\omega_c^2}\frac{\partial^2 y}{\partial t^2} \tag{3}$$

where

$$x = \xi/l \qquad -1 \le x \le 1$$

With the customary assumed product solution, $y(x,t) = Z(x)f(t)$, one arrives at the following ordinary differential equations:

$$\ddot{f} + (3/2)c\omega_c^2 f = 0 \tag{4}$$

$$(1-x^2)Z'' - 2xZ' + cZ = 0 \tag{5}$$

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where c is the separation constant and

$$(\dot{\ }) = \frac{d t}{d t}(\ ) \qquad (\ )' = \frac{d}{dx}(\ )$$

If the separation constant is set to $c=n(n+1)$, the frequency expression as given by Mr. MacNeal results,

$$\omega_n = \omega_c \sqrt{3n(n+1)/2} \tag{6}$$

With $c=n(n+1)$, Eq. (5) has the form of Legendre's differential equation[3] and for $n=1$ the form of the solution is $Z=(\text{const.})x$, i.e. it is satisfied by the pitch (rigid-body) rotational mode. For other values of c, the resulting differential equation may not be satisfied by such a solution.

In Mr. MacNeal's model of the beam it is assumed that the beam has had zero bending stiffness, whereas in the beam model presented in the Note[1,4] it is assumed that the resistance of the beam to transverse loads is due to internal bending moments and the restoring effect of the axial tension due to gravity-gradient is negligible. Thus, the different conclusions reached by Mr. MacNeal and the authors, regarding the lowest vibrational frequencies, are due to the different models used to describe the motion of a thin flexible beam in orbit. In our opinion, a conclusive statement regarding the validity of either model in the limiting situation as the beam becomes slender can only be reached after appropriate in-orbit testing of the behavior of such a structure under (near) zero-gravity conditions. It would appear that as long as the beam offers any bending resistance to transverse loading that the model of Ref. 1 would be valid.

Regarding Mr. MacNeal's comments about the frequencies of parametric excitation, the authors do agree that $\omega_n/\omega_c \equiv 1.0$ is *not* a frequency of parametric excitation. At the center of the first unstable region of the Mathieu chart ($\delta = 1.0$, $\epsilon = 0$), $\omega_n/\omega_c = \sqrt{3}/2$. In Ref. 1 it was the intention of the authors to demonstrate the possibility of instability at very low natural frequency due to small-amplitude pitch motion. It appears that a number of points may be located in the first unstable region for $0<c<0.2$, and $\omega_n/\omega_c = \mathcal{O}(1)$. Figure 2 of Ref. 1 presents a clear example of such an instability. In the tensioned-string model proposed by Mr. MacNeal, a lower limit on the frequency is $\omega_n = 3\omega_c > \sqrt{3}\omega_c/2$. Here again, the difference in results is apparently due to the different models and associated lower bound on the frequencies. If the analysis of Ref. 1 is extended to consider higher order resonances, care would have to be taken to include the correspondingly higher order nonlinear terms in Eqs. (5) and (6).[1] The Mathieu equation (10) is, at best, only an approximation to such a higher order nonlinear system.

References

[1] Kumar, V.K. and Bainum, P.M., "Dynamics of a Flexible Body in Orbit," *Journal of Guidance and Control*, Vol. 3, Jan.-Feb. 1980, pp. 90-92.

[2] Meirovitch, L., *Analytical Methods in Vibrations*, The MacMillan Co., New York, 1967, pp. 161-166.

[3] Spiegel, M.R., *Mathematical Handbook*, Schaum's Outline Series, McGraw Hill, New York, p. 146.

[4] Kumar, V.K. and Bainum, P.M., "Dynamics of a Flexible Body in Orbit," AIAA Paper 78-1418, AIAA/AAS Astrodynamics Conference, Palo Alto, Calif., Aug. 7-9, 1978.

Errata

G80-069 refers to

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Optimal Continuous Torque Attitude Maneuvers

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THE order of the figures on pages 215 and 216 is incorrect. Figures 2a and 2b should be interchanged with Figs. 3a and 3b.

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